Prof. Kincade

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Section 6.1 – Composition of Functions

- $f(g(x)) = (f \circ g)(x)$; $g(f(x)) = (g \circ f)(x)$

"f of g of x"   "g of f of x"

→ How about in real life?
↳ anywhere you've worked a process on something, and then done another working on it.
☆ Take an "x", do something to it, then do something to that newly changed "x".

→ $f(x) = x^2 - 2x + 5$
$g(x) = x + 3$

- $f(g(x)) = f(x+3) \Rightarrow (x+3)^2 - 2(x+3) + 5$
$\Rightarrow x^2 + 6x + 9 - 2x - 6 + 5$
$\Rightarrow x^2 + 4x + 8$

- $g(f(x)) = g(x^2 - 2x + 5) = (x^2 - 2x + 5) + 3$
$= x^2 - 2x + 8$

△ $f(x) = x^2 + 3$
$g(x) = x^2 + 5x - 1$

→ $f(g(x)) = (x^2 + 5x - 1)^2 + 3$
→ $g(f(x)) = (x^2 + 3)^2 + 5(x^2 + 3) - 1$
} Simplify and collect terms

Domain and Ranges of Composed Functions

- $f(x) = x + 7 \rightarrow \mathbb{R}$
$g(x) = \sqrt{x-3} \rightarrow x \geq 3 \not\Rightarrow D = x \geq 3$

"DOES NOT MEAN!!"

$f(g(x)) = f(\sqrt{x-3}) \Rightarrow \sqrt{x-3} + 7$

$x \geq 3 \longrightarrow x \geq 3 \Rightarrow$ Domain $= x \geq 3$

$g(f(x)) = g(x+7) \Rightarrow \sqrt{(x+7)-3} = \sqrt{x+4}$

$x = \mathbb{R} \longrightarrow x \geq -4$

Δ The intersection of all-real-numbers and $[-4, \infty)$ is $[-4, \infty)$!

Therefore, the domain is $[-4, \infty)$

- $f(x) = \sqrt{x+2}$
- $g(x) = 1/x$

$\rightarrow f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}+2}$

$\frac{1}{x}, x \neq 0$

$\frac{1}{x}+2 \geq 0$

$\frac{1}{x} \geq -2$

$1 \geq -2x$

$\boxed{-\frac{1}{2} \leq x}$

$\$$ so, the domain is $[-\frac{1}{2}, 0) \cup (0, \infty)$

$\rightarrow g(f(x)) = g(\sqrt{x+2}) = \frac{1}{\sqrt{x+2}}$

$x \geq -2$

$\sqrt{x+2} > 0$

$x+2 > 0$

$x > -2$

$\rightarrow$ Therefore, the domain is the intersection of $x \geq -2$ and $x > -2$; thus, $(-2, \infty)$

$\rightarrow f(x) = x+2$, $g(x) = x^2-4$ $\Big\}$ $f(g(3)) = f(5) = 5+2 = \boxed{7}$

$(3)^2 - 4 = 5$

Tada!

$\hookrightarrow$ AND/OR $\rightarrow$ $f(g(x)) = (x^2-4)+2 = x^2-2$; $(f \circ g)(3) = 3^2-2 = 9-2 = 7$

- Find $g(f(6))$

$\to g(x+2)$ ~~$= (x+2)^2 + 4$~~ ~~$= x^2+4x+4+4$~~ ~~$= x^2+4x+8$~~ $= (x+2)^2 - 4$
$= x^2+4x+4-4$
$= \boxed{x^2+4x}$

$\to (g\circ f)(6) = (6)^2 + 4(6) = 36 + 24 = \boxed{60}$

OR

$g(\underbrace{f(6)}) = g(8) = 8^2 - 4 = \boxed{60}$

$f(6) = 6 + 2 = \boxed{8}$

#8, 6.1

x	-3	-2	-1	0	1	2	3
f	11	9	7	5	3	1	-1
g	-8	-3	0	1	0	-3	-8

- $g(\underbrace{f(3)}) \Rightarrow g(-1) = \boxed{0}$

  $f(3) = -1$

- $f(\underbrace{f(3)}) = f(-1) = \boxed{7}$

  $f(3) = -1$

- $g(\underbrace{g(1)}) \Rightarrow g(0) = 1$

  $g(1) = 0$

☆ Pg. 406 for graphical method of computing composite functions.

Inverse Functions

☆ One Function "cancels out" another; it undoes the process that another Function Carried out on a number.

→ $f(f^{-1}(x)) = x$ AND $f^{-1}(f(x)) = x$ } These Functions are inverses!!

△ Inverse Functions are simply interchanged inputs and outputs. For example,

$f(x)$:

x	y
1	4
2	5
3	6

⇒ $f^{-1}(x)$:

x	y
4	1
5	2
6	3

☆ NOT EVERY FUNCTION has an inverse!

x	y
1	4
2	4
3	4

⇒

x^{-1}	y'
4	1
4	2
4	3

This is not a one-to-one function because there are multiple inputs and multiple outputs. This is visualized with a horizontal line test!

⇒ $y = 2x - 3$.

① let $f(x) = y$
② Interchange x with y, and vice versa
③ Solve For y

$x = 2y - 3$
$x + 3 = 2y$
$\boxed{\frac{1}{2}(x+3) = y}$ → ④ This is your *potential* new inverse Function.

→ We will test this next time. In the mean time, try this:

If $f(x) = 2x - 3$, and $f^{-1}(x) = \frac{1}{2}(x+3)$, find $f(f^{-1}(x))$ and $f^{-1}(f(x))$. What happens?